# Forward Solver in Magnetoacoustic Tomography with Magnetic Induction by Generalized Finite Element Method

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**Magnetoacoustic Tomography with Magnetic Induction (MAT-MI) is a hybrid imaging modality proposed to reconstruct the electrical impedance property in biological tissue by integrating magnetic induction and ultrasound measurements with high resolution. One of the major problems of MAT-MI is the singularity problem and its numerical errors caused by singular MAT-MI acoustic sources at conductivity boundaries and interfaces. In order to achieve more computational accuracy especially on the conductivity boundaries and interfaces of inclusions, we have developed a forward solver in MAT-MI to compute the problem with Generalized Finite Element Method (GFEM) in the present study. The novelty of the work relies on the first adaption of GFEM in MAT-MI computation. Using the solver the distribution of the eddy current and the distribution of the acoustic source are computed accurately in the object with computer simulation. The results demonstrate the feasibility of the forward solver in MAT-MI. And it shows that it is capable of achieving good accuracy and stability with GFEM.**

***Index Terms*—Biomedical Computing, Bioimpedance, Biomedical Engineering, Computational Electromagnetics, Generalized Finite Element Method, Forward Problem, Acoustic Source Distribution, Eddy Current Distribution, Magnetoacoustic Tomography with Magnetic Induction.**

## I. Introduction

TH electrical impedance imaging of biological tissue has drawn more and more interest in recent years because it has been observed that changes in the electrical impedance are associated with physiological and pathological properties of tissue non-invasively [1]. A hybrid imaging modality named magnetoacoustic tomography with magnetic induction was introduced [2]. It is proposed to avoid the "shielding effect" in Magnetoacoustic Tomography (MAT) and Hall Effect Imaging (HEI) associated with the usage of surface electrodes. It was reported that the spatial resolution of MAT-MI is better than 2 mm in reference , which is higher than Electrical Impedance Tomography (EIT) [3] and Magnetic Induction Tomography (MIT). Although Magnetic Resonance Electrical Impedance Tomography (MREIT) can achieve high spatial resolution, it is currently limited by its requirement of high level current injection.

One of the major problems of MAT-MI is the singularity problem and its numerical errors caused by singular MAT-MI acoustic sources at conductivity boundaries and interfaces. The calculation of MAT-MI acoustic source at conductivity boundaries and interfaces was either approximated by a linear interpolation or ignored. To avoid the singularities in the forward calculation of the MAT-MI acoustic sources, an integral forward method in a finite volume was reported. Such singularity makes the computed MAT-MI acoustic sources at conductivity boundaries and interfaces not convergent when conventional Finite Element Method (FEM) is utilized.

In this work, we address the problem for calculating MAT-MI acoustic sources accurately and stably. In our simulation, the models with GFEM show very good convergence properties when solving the problems with singular solutions in MAT-MI. The simulation results demonstrate the feasibility of the model in MAT-MI forward solution.

## II. Method

The physical process of obtaining the induced ultrasound signal by the stimulating magnetic fields is defined as the forward problem for MAT-MI. In the forward problem, electromagnetic energy with high frequency is injected into the object and induces detectable ultrasound. As illustrated in Fig 1, pulsed electromagnetic stimulation $B_1$ is sent out by the excitation coils and eddy current is induced in the conductive object volume. The corresponding current density distribution $J$ is determined by both the stimulating magnetic field and the conductivity distribution $\sigma$ . Given the static magnetic field flux density $B_0$ , the Lorentz force exerts on the eddy current and then acts as a source of acoustic waves which can be measured by transducers around the object.

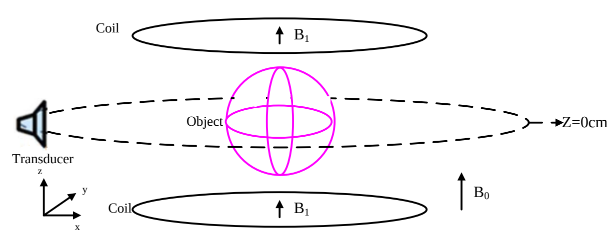


Fig. 1. Illustration of MAT-MI forward problem.

The wave equation governing the pressure distribution is given in Equation (1) as in [2]

$$\nabla^2 p(r,t) - \frac{1}{c_s^2}\frac{\partial^2 p(r,t)}{\partial t^2} = \nabla \cdot (J \times B_0) \qquad (1)$$

where $p$ , $r$ , $t$ and $c_s$ represent the pressure, the spatial point, time and the acoustic speed, respectively. The MAT-MI acoustic pressure obeys wave equation

$$p(r,t) = -\frac{1}{4\pi}\oint_V dr'\nabla\cdot[J\times B_0]\frac{\delta(t-R/c_s)}{R} \quad (2)$$

where $r'$ is the location of acoustic source, $R=|r-r'|$, $V$ is the volume containing the source and $\delta$ is the delta function.

For numerical calculation of the MAT-MI acoustic source in a piecewise homogeneous conductivity distribution, the gradient of conductivity is singular at boundaries between pieces of different conductivity media.

The GFEM is a case of the partition of unity method [4-8]. It can have more than 2 or 3 generalized degrees of freedom. Suppose $S^h$ is the conventional FEM space and $[\varphi_1\ \varphi_2 \cdots \varphi_n]^T$ is the shape function. The field variable $U^h$ can be written as a summation with the conventional FEM:

$$U^h = \sum_{i=1}^{N} \vec{u}_i \vec{\varphi}_i \quad (3)$$

where $\vec{u}_i (i=1,2,\cdots,N)$ is the vector of degrees of freedom on the $i^{th}$ node which represents the potential variation on the node [9]. We choose a singular $\vec{\varphi}_i$ as the shape function, mimicking the singularity of the solution. Otherwise, a Lagrange interpolation is used as the shape function to calculate the MAT-MI acoustic sources without singularities. And the method of weighted residuals is used to derive the governing equations of the GFEM. The boundary integral is carried out for conductivity interfaces.

## III. Simulation And Results

A numerical spherical model is built and the first-order GFEM is adopted. The calculated magnetically induced eddy current in normal direction and the corresponding simulated vibration acoustic source in $Z=0$ plane are shown in Fig 2 (2) and (3), respectively. Fig 2 (4) shows the estimated pressure with the transducer location at point (-0.14,0,0).

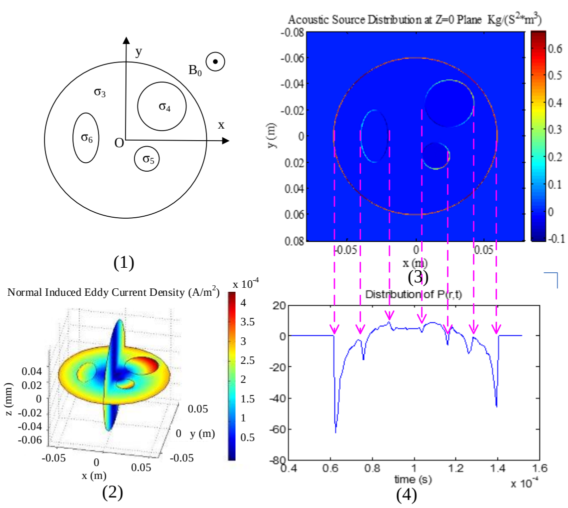


Fig. 2. (1) Numerical Model, (2) the Distribution of Normal Eddy Current, (3) the Acoustic Source and (4) the Estimated Pressure.

The eddy current has large amplitude flowing in the high conductive inclusion (such as breast tumors) regions as compared to their surrounding regions. The distribution of acoustic pressure in spatial domain has a close relation to the conductivity distribution. And the sequence of the acoustic pressure calculated on the detection point is sensitive to the change of conductivity distribution inside the object. It indicates that the detected acoustic signals contain information for inverse conductivity reconstruction. And the order of pressure is in the detectable range of current commercial transducers as shown in Fig 2 (4).

## IV. Conclusion

With the developed method, the generalized convergent forward solution of MAT-MI has been obtained at any locations in the entire computation domain of the object. And it is well prepared to solve the inverse problem. In summary, the model with GFEM can provide an accurate and promising solution in forward problem of MAT-MI.As shown in the results, in MAT-MI, the tissue with higher conductivity value produces higher strength of acoustic source and signal, so it plays a bigger role in the pressure generation of MAT-MI. It suggests that MAT-MI has the potential to locate the tissue, organs or tumors, which have lower impedance (high conductivity) value than their surrounding regions.


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